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Economic theorists have often concentrated on global issues of economic development: the long run path of per capita incomes, the existence and character of balanced growth, the intertemporal optimality of alternative growth trajectories. This has led to macroeconomic theories characterized by a few relatively simple, but dramatic properties such as the "iron law of wages" that derived from classical reasoning, or the currently fashionable "golden rules of economic growth." But policymakers and the rank and file civil servants who are charged with implementation, have long known that an awareness of the "big issues" is not sufficient by itself to guide the host of individual decisions for which they are directly responsible or over which they hope to hold sway through well conceived direct and indirect controls. They have found that sooner or later policies must account for the realities of decision-making in the field and factory. Unfortunately for them, however, at this microeconomic level, little guidance can be obtained from the traditional economic literature. There has been a wide gap between the principles of macroeconomic development theory and the practice of policy makers and administrators.

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This study attempts to help fill this gap by developing and testing a dynamic, microeconomic model that is capable of simulating the performance of an individual sector, in this case agriculture, in a way that explicitly accounts for various strategic details of technology and decision-making. Our first purpose has been to improve our understanding of the development process. Our second purpose is to aid the formulation of effective development policy by making possible detailed projections and comparative dynamic analyses of proposed governmental policies at the intra-sector level.

Part 1 of our paper outlines the general requirements for a dynamic, microeconomic model of agricultural development. Part 2 then presents a mathematical theory that incorporates what we think are the essential features or strategic details of the process. In part 3 this theory is approximated by an operational model that can be estimated and simulated within existing data and computational limitations. Part 4 is devoted to testing the model's ability to describe recent agricultural history in the Central Punjab of India. We find the model performs fairly well and suitable modifications should be applicable to virtually any region undergoing a transition from traditional to modern agriculture.

1. THE STRATEGIC DETAILS OF DEVELOPMENT¹

1.1. The peasant farmer as an "economic man".

Until recently it was argued by many, and with great force, that people in various societies behave according to rules so different that microeconomic theory is not relevant, that the people of less developed countries are tradition bound, that cultural and institutional restraints severely circumscribe their responsiveness to market incentives, and that the developed

countries have a kind of monopoly on "economic man."² SCHULTZ [1964] on the other hand argued that traditional patterns were maintained not because of hidebound restraints but because they represented a rational equilibrium under existing conditions. His position has been confirmed by the growing number of supply response studies in the LDC's.³

Focusing on the question of whether or not peasants in traditional or near traditional agriculture respond to opportunities which are made available by changes in market conditions, various investigators have shown that agricultural production is price responsive, especially when adjustment lags due to uncertainty and quasifixity of capital stocks are accounted for. Moreover, they suggest that the general form and direction of this response is consistent with price theory and that peasants in traditional agriculture respond to market incentives when sufficient incentives exist.

It is on the basis of these results that we believe behavior of farmers in the LDC's can be represented by a model in which choices among well defined alternatives are made by explicitly attempting to maximize the attainment of well defined goals. It seems, however, that the conventional marginal analysis does not adequately describe maximizing by peasant farmers as it really occurs. We think that at least six complications should be incorporated into the analysis. These are the interdependence of farm household and firm decisions, multi-product, multi-process technology, uncertainty, technological change, learning and nonfarm linkages. We shall comment briefly on these in turn.

1.2. Interdependence of farm household-firm decisions.

Economists have traditionally simplified the overall economic allocation problem into two separate parts: the household income allocation problem, described by constrained utility maximization, and a firm resource allocation

problem described by profit maximization. Nowhere is this theoretical tactic more clearly expounded than in KOOPMANS [1957] where the principles are illustrated with the "time-honored example of a man by whom production and consumption decisions are made in combination: Robinson Crusoe..." who in the course of the analysis is shown to be decomposable into Robinson the producer and Robinson the consumer.

Crusoe is not merely a convenient literary illusion. He is the prototype of the "peasant" or "family" farmer found in virtually every agricultural region in the world. But while, for the sake of simplicity, the farm decision is no doubt broken up into smaller, more manageable parts in practice and while we shall indeed exploit a given decomposition hypothesis below, it does violence to reality to suppose that the decomposition takes place on the farm as it does in the nonfarm economy. Some authors have recognized the fundamental interdependence in the farm between firm and household decisions. HEADY, BACK and PETERSON [1953] were among the early investigators to quantify this interdependence. More recently NAKAJIMA [1957A, 1957B and 1965] and MELLOR [1965A, 1956B] have contributed to a clearer theoretical understanding of this interdependence in the context of the less developed countries. It is now time to incorporate this feature in an empirical model of production response in traditional agriculture. KRISHNA [1965] has made a step in this direction by deriving a marketable surplus supply function from a mathematical version of Nakajima's analysis. Our model represents another, somewhat more elaborate step.

1.3. Farm technology

The neoclassical analysis of the firm is for the most part based on twice differentiable production functions which are usually assumed to involve a single output and which represent a given production technology.

Contrastingly, agriculture is really characterized by multiple outputs, and during periods of transition (which constantly occur), by multiple technologies. Activity analysis, as developed by KOOPMANS [1951], LEONTIEF et al [1953] and applied by many investigators can accomodate all three of these characteristics in any amount of detail.

Direct observation leads us to appreciate the fact that traditional agriculture is a complex phenomenon with hundreds of individual tasks being performed, in many possible combinations, requiring detailed knowledge of soils, climate, topography, and with scarce resources being distributed over time and crop use. Choices among these many tasks are merely enlarged when new implements, power sources, and materials are introduced. We do not argue that it is necessary for the purposes of development policy to accomodate all of the details with which the peasant himself must contend. We do believe that many of them are important and that only by representing major technological alternatives in an activity analysis framework can agriculture be effectively understood and planned -- at any level.

1.4. Uncertainty

The fact that farming is highly uncertain in many of its aspects is obvious to a casual observer. Accounting for it in some way is a virtual necessity for the farmer and if he is to understand agriculture a necessity for the economist as well. It seems doubtful, however, that the farmer's decision strategies are the same as those used by sophisticated gamblers in St. Petersburg or Monte Carlo. It seems likely instead that his strategies come closer to rules that might be summarized as strategies of cautious optimizing. Examples are the behavioral bounds of CYERT and MARCH [1963], the focus-loss principle of SHACKLE [1958], the chance constrained programming models of CHARNES and COOPER [1959], and the

safety first principle of ROY [1952]. We have taken this latter point of view and as a first approximation, have adopted a particular representation of it elaborated by one of us elsewhere, DAY [1971].

1.5. Technological change

The principles just outlined when properly constructed would be quite consistent with, indeed would help explain a state of economic equilibria in traditional agriculture, a state according to SCHULTZ [op. cit.] in which, given the state of the arts, the rates of return to traditional inputs are so low that little or no net investment takes place, and in which comparatively few significant inefficiencies in the allocation of the factors of production exist. In such a state he argues small changes in either the relative prices of inputs or in the quantities of inputs unchanged in quality are unlikely to bring about any long run departure from this equilibrium. As a result, only new technology can shift agriculture from this traditional state.

Within the activity analysis framework at least four specific components of "new" or nontraditional technology should be considered: new materials, new implements and power sources, and new cultural practices. Activities involving these and traditional activities, accommodated within the set of possible farm operations enable the many choices describing the transition from traditional to modern agriculture to be analyzed.

1.6. Learning and adoption

The breakdown of age old practices takes time partly because the supply of new inputs must go through a development process of its own. This places external constraints on adoption of new technology, a factor no doubt of great importance. In addition, adoption is internally constrained by a learning process which proceeds as more and more farmers gain familiarity

with and confidence in their ability to successfully exploit the new opportunities. The impact of new technology, following upon its innovation is thus distributed over time, a fact that should clearly be a part of a complete analysis of development, and a further complication to be incorporated in a model of an agricultural region based on the principle of economizing.

1.7. Nonfarm linkages

We have mentioned the external constraints imposed by limited supplies of nonfarm inputs such as industrially produced implements, machines, materials, and fuel. This means that development takes place within a multi-sectoral context. Several additional nonfarm linkages are crucial. These involve the supply of credit, the supply of wage-labor, and the demand for final products. Some of these linkages occur indirectly through market prices, and some occur directly through physical and behavioral limitations on the use and availability of resources. Hence, even in models that focus almost entirely on development and planning within the sector these strategic linkages must be accounted for.

2. THEORY

It should now be clear that a complete understanding of agricultural development must involve first, an analysis of how development takes place within the farm sector, and second, a multisectoral analysis of economic development as a whole. It is beyond the scope of the present undertaking to meet both of these requirements. We concentrate here on the first of these, developing an adaptive, multi-goal theory of decision-making that serves as a guide to the construction of an operational farm sector model.

2.1 Decisions

2.1.1 Farm Activities

Farm activities include production, sales, investment, financial and household activities. Let X be the complete set of these activities. We shall denote an activity by its name or index and equate X with the set of such names or indexes. Hence $j \in X$ denotes the name or index of activity j . The intensity with which a given activity is operated we call an activity level and denote it x_j , $j \in X$. The units depend on the activity in question. Most production activity levels are measured in acre units, others are in units of volume or weight, some in monetary units. All of them indicate the planned intensity to be operated with in a given year with the plans drawn up at the beginning of the year. The decision vector $x = (x_j)_{j \in X}$ is the n -vector of activity levels.

The choice of farm activities for a given year is constrained by three categories of relations: technological, financial and learning. The first category involves labor, land, commercial input and machine capacity constraints. These define a set T of technologically feasible decision vectors. The second category involves working capital and availability, borrowing limitations, and debt repayment requirements. These define a set F of financially feasible decision vectors. The third category represents the constraining effects of learning on the adoption of new techniques and leads to a set L of decision vectors compatible with learning. We shall describe the specific structure of these sets in section 3. At this point we need to recognize their dependence on data germane to the decisions for a given year. Each set depends on two types of coefficients which we call constraint and limitation coefficients. Denoting these by vectors B and c respectively, we write

$$(1) \quad T = T(B^T, c^T)$$

$$(2) \quad F = F(B^F, c^F)$$

$$(3) \quad L = L(B^L, c^L)$$

Letting $B := (B^T, B^F, B^L)$ and $c := (c^T, c^F, c^L)$ the region of feasible decisions for a given production period may be denoted

$$(4) \quad \Gamma(B, c) := T \cap F \cap L$$

It is the set of decision vectors that satisfy simultaneously the technical, financial and learning constraints.

2.1.2 Farm Goals and Lexicographic Utility

We assume that the farm has four goals arranged in an absolute priority order. We assume also that these goals can be represented by four real-valued objective functions $\mu_i: X \rightarrow \mathbb{R}$ where $\mu_i(x, a^i)$ gives the level of satisfaction of the i^{th} goal given by the decision vector x and where a^i is a vector of parameters. These goals are

μ_1 = the goal of satisfying subsistence consumption;

μ_2 = a goal representing a preference ordering
amongst alternative current cash consumption
and future forecasted income streams;

μ_3 = a metric defining the distance of a given
choice from a set of safe-enough choices Z ,
i.e., $\mu_3(x) = -\text{dist}(x, Z)$;

μ_4 = net cash returns or profit function.

We let $\alpha^i \in \mathbb{R}$ be a "satisficing level" for the i^{th} goal and let

$\varphi_i: X \rightarrow \mathbb{R}$ be defined by

$$(5) \quad \varphi_i(\cdot, a^i, \sigma^i) = \min\{\mu_i(x, a^i)\sigma^i\}, \quad i = 1, \dots, 4.$$

We now suppose that the farmers' plans can be represented by the maximizing $\varphi_1, \dots, \varphi_4$ in priority order, subject to technical, financial and learning constraints.⁴

The first goal seems reasonable, and relevant in regions where a major part of production is produced for home consumption. The second goal is a device for simplifying the total decision problem. It is structured so as to represent the allocation of cash resources between consumption and saving. The optimum allocation of cash saved amongst farming and financial activities is then determined by maximizing goal 4, while the optimum allocation of consumption expenditures amongst individual items is assumed to be determined by maximizing a fifth objective function unspecified in this study.

Goal 3, the safety metric, represents behavior according to a principle of cautious optimizing very much like the safety-first principle or chance-constrained programming. It is more general than those, in that it does not require the specification of any subjective probabilities. It covers unpleasant contingencies other than those covered by the subsistence goal and is meant to represent a strategy to protect the farmer against uncertain but highly damaging feedback effects of extreme departures from previously experienced and successful behavioral patterns. Of course if behavior to guarantee subsistence requires it, extreme departures from past experience are predicted according to the maximization of the subsistence goal. However, given satisfaction of the first two goals, caution plays a role in limiting response to shortrun profit opportunities as incorporated into goal 4.

2.1.3 Farm Decisions as an L* Program

These hypotheses amount to maximizing a Lexicographic* or L* utility function subject to constraints. Let $X_0^* = \Psi_0(B, c) := \Gamma(B, c)$. Then we define the L* programming problem (ROBINSON and DAY [1970]) to be the sequence of maximization problems

$$(7) \quad \pi_i(a', \sigma', \dots, a^i, \sigma^i, B, c) := \max_x \{\varphi_i(x, a^i, \sigma^i) \mid x \in X_{i-1}^*\}, \quad i=1, \dots, 4,$$

where

$$(8) \quad X_i^* := \Psi_i(a', \sigma', \dots, a^i, \sigma^i, B, c) := \{\varphi_i(x, a^i, \sigma^i) \geq \pi_i\} \cap X_{i-1}^*,$$

$i=1, \dots, 4$

is the set of choices maximizing the i^{th} goal given that they are feasible and that they maximize (or satisfy) the higher order goals.

2.1.4 Super Utility

This scheme (7)-(8) is called a fourth order weak L* program. (ROBINSON and DAY [ibid].) It has been shown elsewhere, that this decision-making procedure is equivalent to ordinary constrained utility maximization in the following sense. given certain conditions there exists a super-utility function say $\varphi: X \rightarrow \mathbb{R}$ with parameters $a := (a^1, \sigma^1, \dots, a^4, \sigma^4)$ such that the set of solutions

$$(9) \quad X^*: \Psi(a, B, c) := \{x \mid \varphi(x, a) \geq \pi\} \cap \Gamma(B, c)$$

to the ordinary program

$$(10) \quad \pi(a, B, c) := \max_x \{\varphi(x, a) \mid x \in \Gamma(B, c)\}$$

is exactly $\Psi_4(a, B, c)$ of (8).

This super-utility function represents a preference ordering over activities which accounts for all of the farmers' considerations, in so far as they affect his behavior, of subsistence, commercial consumption, safety and profit goals, in their priority order. This function is

probably too complicated to use operationally and we use the L^* approach instead as a guide to constructing the operational model. However, it is notationally convenient for summarizing the complete model and we use (9) in what follows with the understanding that it is meant as a representative of (7)-(8).

In theory the set $\Psi(a,B,c)$ is in general non-unique. While selection amongst these possible best choices could be explained by a variety of plausible hypotheses we use here algorithmic selection, i.e. choice determined by the first point in Ψ obtained by our computer code. Since in fact Ψ is often single-valued this is not necessarily a restrictive assumption. However, to complete the model we must define a selection operator, we denote it R so that the theoretical prediction of farm plans in a given year is

$$(11) \quad x^* = R \cdot \Psi(w) \quad \text{where } w = (a,B,c) .$$

Since the realized data $w = (a,B,c)$ may change in the very shortrun (from month to month or even more often) plans may in reality be modified during the crop year. We have not yet tried to account for such shortrun planning revisions in our model, but have instead used x^* as defined in (11) as our estimate of actual behavior.

2.2 Feedback and the Complete Model

2.2.1 Feedback

The data vectors (a,B,c) on which depend decision vectors for a given year depend themselves on previous decisions, previous data and on exogenous variables linking the farm situation to its "external" environment.

Satiation levels σ^1 and σ^2 may depend on past subsistence and commercial

consumption activity levels while the desired safety level σ^3 may depend adaptively on new information on price and income variability. Resource limitations c^T depend on past investment activities, while financial bounds depend on previous expenditures, and borrowing activities. Learning proceeds with experience so that learning limitation coefficients c^L may depend on previous utilization of "new" technologies. Price expectations based on lagged pricing enter the profit objective and in various coefficients of the financial constraints. Other variables representing the state of the outside economy may be included in calculations of relevant planning data. These observations lead us to recognize a feedback effect of past behavior on current plans and the linkage of farm sector to the nonfarm economy. The structure of this feedback, as we have modeled it in the current study, is now described in abstract terms.

2.2.2 Data Feedback Structure

We date plans and data with a time subscript to indicate the beginning of the year in which they are determined. Hence x_t^* is the solution to the L^* program (7)-(8) and w_t is the vector of all the data on which it is based. Hence we rewrite (11) as

$$(12) \quad x_t^* = R \cdot \Psi(w_t) .$$

To define the adaptive dependence of the current data on past plans and past data we adopt the following convention. Let v_t be an arbitrary n -dimensional vector. Then

$${}_s v_t = (v_s, v_{s+1}, \dots, v_{t-1}, v_t)$$

is a $(t-s+1)n$ -dimensional vector with $t-s+1$ component n -vectors. The adaptive feedback effects summarized in § 2.2.1 can now be represented

by the expression

$$(13) \quad w_t = \omega(t-\tau^x_{t-1}, t-\tau^w_{t-1}, z_t)$$

where z_t is a vector of exogenous variables not explained by the theory but representing linkages with the nonfarm sector (and possibly including lagged exogenous variables) and where ω is a vector of functions each element of which defines the dependence of one data parameter on past decisions and exogenous variables. Of course many if not most of these will be constant functions, meaning that the coefficients are constant. But the notation is general enough to accommodate many types of realistic feedback effects and outside influences.

2.2.3 The Complete Model

Our theory which is concisely summarized by equations (12) and (13) yields a discrete time, open dynamic system consisting of a set of simultaneous τ^{th} order difference equations of a complicated and highly nonlinear nature. It represents current decisions by a decision operator depending on considerations of technology, finance, learning, subsistence, commercial consumption, safety and profits. This decision operator involves choosing amongst feasible alternatives according to a hierarchy of goals on the basis of data that depends on previous decisions and outside influences. It is a microeconomic theory of farm behavior that incorporates in a theoretically consistent manner the strategic details of farm development for which it was our purpose to account.

2.3 Aggregation

We have gone to the trouble of constructing a theory of farm behavior because we felt that on a detailed understanding of their behavior would depend an adequate explanation of economic development in the sector as a

whole. Obviously, however, it is impossible to derive regional aggregates by adding up predictions for each farmer. Instead, we use the structure specified by (12)-(13) to define a regional model to be used for explaining and projecting various regional variables.

The theory of aggregation required to go from the micro level to a regional aggregate is complex and only partially developed. (DAY [1963].) It cannot be gone into here. We proceed, however, on the following assumptions. Let w_t^i , x_t^{*i} , Ψ^i , ω^i be the data vector, the plan, the decision operator and the feedback operator respectively for the i^{th} farmer. We assume that there exists regional data vectors W_t , Z_t that can be obtained from a suitable aggregation of individual farm data and that there exist regional decision and feedback operators Ψ and ω such that the regional analog of (12)-(13) given by

$$(14) \quad X_t = R \cdot \Psi(W_t)$$

$$(15) \quad W_t = \omega(t-\tau X_{t-1}, t-\tau W_{t-1}, Z_t)$$

possesses the following aggregation property

$$(16) \quad X^t = \sum_i x_t^{*i}.$$

Such a region is aggregatable and allows individual decision units to be subsumed. The assumptions required for (16) to hold are very strong and would not be true necessarily even if the theory behind equations (1)-(13) were exactly true -- which it is not -- for each farm. Consequently, a model based on (14)-(16) can at best only be an approximate theory of behavior at the sector or regional level.

2.4 Implications

Before turning to empirical matters let us pause to consider how this theory represents the development process. Given initial conditions of low or nonexistent capacity in highly productive technology, farm behavior will be dominated by subsistence goals. If external demand conditions and internal productivity permit it, commercial sales will lead to cash income for which consumption and farm investment will compete. Subsistence considerations will be gradually pushed into the background. As cash farming grows in importance caution in response to market forces and profit maximizing will come to dominate farm production and investment decisions. Depending on the initial situation farmers might adopt new technology rapidly or in some cases not at all. Indeed, many alternative histories are possible in such a model, with many different phases or stages of development arranged in many possible alternative sequences.

Equilibrium at a stationary state might come about in the absence of technological change, though nothing in the theory guarantees that possibility. Indeed such empirical evidence as we now have suggests that agriculture in very diverse situations is inherently unstable once commercial farming activities become important. (HEIDHUES [1966], MUDAHAR [1970] and MUELLER [1970].) The cause seems to lie in the highly inelastic demand for agricultural produce and its feedback effect through price and working capital supplies.

The investigation of the existence of stationary states, their stability or instability, and the possibility and character of multiple phase and even indeterministic solutions to theoretical systems of the type (15)-(16) has been begun and the interested reader is referred elsewhere for a further discussion of these matters, DAY and TINNEY [1969], DAY and KENNEDY [1970].

However, one property of this theory of such great importance that we

should comment on it before proceeding, is its incomplete determinacy in the following sense. We specified a selection operator which we acknowledged to be more or less arbitrary: after the goals that rationally might be pursued in the L^* program there remains an indeterminate residuum of choices contained in the image of Ψ . Even if this set contains more than one member only infrequently (as we suggested would be the case) the element of incomplete causality clearly remains. The implication is that from time to time decision-makers' choices may be arbitrary -- perhaps random or unpredictable -- and hence the evolution of society imperfectly predictable as well. At best society's behavior would seem to be predictable within bounds.

On the basis of this consideration we should be highly surprised if our operational model predicts actual history with extreme accuracy. This causal incompleteness is fundamental to the theory and not the result of aggregation errors due to the failure of the assumptions behind equations (14)-(16). Adding the latter source of error to the former we are led to take the position that approximate accuracy of our model in explaining the past is a very strong confirmation of its fundamental validity, just as it is insufficient grounds to believe that projections based on it will have more than a crude (though perhaps highly valuable) contribution to policy.

3. AN OPERATIONAL FARM SECTOR MODEL

We now outline our initial approximation of the theory just developed. A detailed exposition is in SINGH [1971].

3.1 Feasible Decisions

Activities are assumed to be linear, finite in number and their levels X_j , $j \in X$ are measured for the regional aggregate. Constraining factors are identified by an index $i \in Y$. The technical coefficients b_{ij} , $i \in Y$, $j \in X$

are assumed constant over time and all technology is assumed to be embodied. Positive (negative) coefficients mean a given factor in a net input (output); a zero coefficient indicates a factor not involved in the activity in question. Limitation coefficients C_i , $i \in Y$ and also defined at the regional level; positive (negative) coefficients are associated with upper (lower) bounds on activity combinations, zero coefficients with balance constraints.

Production activities, $j \in P$, include land preparation, planting, cultivating, fertilizing, harvesting, processing, and transporting. These are distinguished where relevant by type of soil, by type of technology (irrigated, unirrigated, fertilized, unfertilized, bullock, tractor, etc.), by crop and by season (summer and winter). Household activities, $j \in M$, include subsistence, food consumption, commercial consumption, and labor "supplying" on and off farms. Purchase activities, $j \in B$, include the purchase of variable inputs such as fuel, fertilizer, improved seeds, etc. Investment activities, $j \in I$, include land development and the purchase of capital goods such as tractors, motors, implements, bullocks, camels, etc. Financial activities, $j \in F$, include saving, borrowing and debt repayment. Sales activities, $j \in S$, are included for each commercial crop.

Labor constraints, $i \in W$, include exogenous supplies of village wage labor, regional labor and national labor. These supplies are to be augmented by household activities which supply family labor in various amounts by season. Farm family labor supplies are limited exogenously in the current model by the number of farm families, though we hope in the future account for these variables endogenously. Material constraints, $i \in E$, allow for the exogenous specification of regional supplies of electricity, fertilizer, herbicides and insecticides limiting material purchase activities.

Land supplies, $i \in L$, in some categories can be augmented by investment in land development (irrigation, drainage, etc.) but total amounts available are constrained by overall regional supplies. Machinery, $i \in M$, is limited by inherited (depreciated capacity) but can be augmented by investment. Finally output balance constraints, $i \in O$, connect the production of commercial crops to the sales activities of the farm. These constraints together with nonnegative restrictions approximate the set of technologically feasible activities, T , of equation (1) § 2.1.1.

Household activities involving commercial consumption material and labor purchases and investment activities all compete for working capital. Financial activities involve additions to working capital through borrowing or deductions through debt repayment or cash savings. The former are limited by external banking rules, the latter by borrowing and cash commitments. These involve a set of linear constraints, $i \in F$, that approximate the set F of financially feasible farm activities of equation (2) § 2.2.1.

We have emphasized the role of learning on the part of farmers in transition and argued that the learning process limits the speed of adoption of new inputs, outputs, or production practices. In a given year a set of adoption constraints, $i \in N$, limit activities that involve these new things. These upper-bounding constraints approximate the set L of equation (3), § 2.2.1.

With these definitions we obtain the polyhedral approximation of the theoretical set of feasible regional aggregate decisions for a given year:

$$(4) \quad \tilde{\Gamma}(B, C_t) = \{X \mid \sum_{j \in X} b_{ij} X_j \leq C_{it}, i \in I, X_j \geq 0, j \in X\}.$$

3.2 Goals and Satisficing Constraints

3.2.1 Subsistence

Subsistence activity is the result of a combination of physiologically determined needs and socially conditional wants. Various household activities satisfy these needs and wants in varying degrees. In this study we have not yet modelled these details explicitly but, assuming the satiation of ϕ_1 throughout the period we have estimated lower bounds on subsistence consumption of each of several crops (wheat, maize, rice, sugarcane and pulses) exogenously using data from farm budgeting studies.⁵

3.2.2 Commercial Consumption -- Cash Saving

In this initial study cash expenditure on consumer goods is treated as an exogenous variable, \bar{E}_t , and is deducted from sales revenue of the preceding period to arrive at the initial working capital supply for a given year. This is equivalent to assuming that the cash consumption goal, as represented by utility⁶ function ϕ_2 , is satiated throughout the period.

3.2.3 Safety

The safety metric can be introduced as a fundamental axiom of behavior DAY [1971] or it can be derived from the safety-first ROY [1952] or focus-loss SHACKLE [1958] principles of decision making under risk BOUSSARD [1969], PETIT and BOUSSARD [1967]. When the safety goal is satiated the metric circumscribes decisions by an ellipsoidal "Safety Zone" which can be approximated by supporting hyperplanes as in the case of the subsistence goal.⁷ In this study we assume satiation and approximate the implied safety-zone by three sets of linear inequalities. The first two are sets of upper and lower bounds on crop acreages which prevent extreme changes in cropping patterns and protects the farmer against a drastically changed pattern of

relative profitabilities of cash crops at the end of the season. The second prevents extreme increases in capital stocks and protects the farmer from sinking too much capital in one opportunity when another, perhaps currently unknown one, may be more desirable in the future.

The set of upper and lower bounds on individual cash crop acreages may be denoted as follows: the upper bounds are

$$(18) \quad \sum_{j \in P_i} X_{jt} \leq C_{it}^u, \quad i \in S$$

where S is the set of cash crops and P_i is the set of harvesting activities for crop $i \in S$ whose levels are measured in acres; the lower bounds are

$$(19) \quad \sum_{j \in P_i} X_{jt} \geq C_{it}^l, \quad i \in S$$

The constraints in the third group are defined for each activity involving investment in some capital good. Let M be the indexes of such goods and let j' indicate the investment activity associated with good $j \in M$. Then the investment bounds for each year are

$$(20) \quad X_{j',t} \leq C_{jt}, \quad j \in M$$

The set of activities that satisfy the several safety bounds may be defined as

$$(21) \quad Z_t = \{X_t \mid \sum_{j \in X} b_{ij} X_{jt} \leq C_{it}, \quad i \in Z\}$$

where Z is the set of "safety factors" and where C_{it} , are the upper and lower bounds of (18)-(20).

3.2.4 Profits

The anticipated costs and returns of particular activities that enter the profit objective fall into four classes: expected returns from sales activities $a_j^t \geq 0$, $j \in S$, current costs of purchasing activities $a_j^t \leq 0$, $j \in B$, an annual depreciation charge $a_j^t < 0$, $j \in M'$ that must be recovered to justify an investment, and interest costs and returns associated with borrowing activities. If \hat{p}_i^t is the anticipated unit price of crop i then $a_j^t = \hat{p}_j^t$, $j \in S$. Likewise, if p_i^t is the current price of inputs then $a_j^t = -p_j^t$, $j \in B$. In the case of investment goods the depreciation charge based on straight line depreciation, is the current investment good price p_i^t divided by the average life λ_i , $i \in M$. Hence, $a_j^t = -p_j^t/\lambda_j$, $j \in M$. In the case of borrowing a_j is equal to the negative of the average interest rate in that category of loans. For saving it is the positive average bank rate on time deposits. In order to account for strong liquidity preference we include a transfer activity of working capital to the farm "investment account" at a cost determined by an internal risk premium. This premium is computed so that farm investment will occur only if its pay back period is five years or less. All other a_j coefficients are zero.

With these several assumptions the profit objective defined at the regional level corresponding to the utility function, ϕ_4 , becomes

$$(22) \quad \phi_4(X_t, a^t) = \sum_{j \in X} a_j^t X_j^t.$$

The satiation level for this function is assumed to be unbounded (i.e., $\sigma^4 = \infty$).

3.3 The Approximating Linear Program

Assuming that the high order goals ϕ_1, \dots, ϕ_3 can be satiated during the period under study the set of feasible farm plans approximating X_3^* of

equation (8) is that set satisfying the collection of linear constraints (4'), (17)-(21). We denote this set

$$(23) \quad \Lambda(\tilde{B}_t, \tilde{C}_t)$$

in which \tilde{B}_t is the complete set of coefficients entering all of the constraints (4'), (17)-(21). Hence, the set of farm plans intended to approximate (9) at the aggregate level is the set of solutions to the linear programming problem:

$$(24) \quad \tilde{\pi}(\tilde{a}_t, \tilde{B}_t, \tilde{C}_t) := \max_{X_t} \{ \langle \tilde{a}_t, X_t \rangle \mid X_t \in \Lambda(\tilde{B}_t, \tilde{C}_t) \}$$

We denote this set

$$(25) \quad \tilde{\Psi}(\tilde{a}_t, \tilde{B}_t, \tilde{C}_t) := \{X_t \mid \langle \tilde{a}_t, X_t \rangle \geq \tilde{\pi} \cap \Lambda(\tilde{B}_t, \tilde{C}_t)\}$$

It is our initial operational analog of equation (14).

3.4 Feedback and Exogenous Variables

The data vector $(\tilde{a}_t, \tilde{B}_t, \tilde{C}_t) = \tilde{W}_t$ contains both exogenous variables and variables generated by explicit feedback functions representing physical accumulation or adaptive behavior. We now outline the specific assumptions used here.

3.4.1 Labor and Materials

Variable inputs including labor and materials are (except for family labor) assumed not to be inventoried on the farm so that needed supplies must be purchased by the appropriate purchasing activity. The materials constraints are divided into two groups, a set of balance constraints with indexes E_1 representing the purchase requirements placed by farm demand and

a set of purchasing restrictions with indexes E_2 that limit purchases to exogenously given supplies. Then $C_{it} = 0$, $i \in E_1$ for all t and $C_{it} = Z_{it}$, where Z_{it} is the exogenously given supply of input $i \in E_2$. In some cases we have assumed that C_{it} is not limiting, such as electricity and regional labor. In other cases we estimated a finite magnitude such as family and village labor.

3.4.2 Capital Goods

Stocks of land by type based on soil classification, and seasonal availability were treated exogenously, as was the supply of land that could be irrigated by canal. Hence we have $C_{it} = Z_{it}$ when Z_{it} is the exogenous supply of land of type i , $i \in L$.

Machinery utilization constraints are generated endogenously by a "trapezoidal" depreciation formula

$$(26) \quad C_{jt} := C_{jt-1} - \frac{2}{\lambda_j} \sum_{s=\ell_j}^{\lambda_j} b_{jj'} X_{j,t-s}^*, \quad j \in M$$

where λ_j is the average life of machine j and where

$$\ell_j = \begin{cases} \lambda_j/2 & \text{when } \lambda_j \text{ is even} \\ (\lambda_j + 1)/2 & \text{when } \lambda_j \text{ is odd.} \end{cases}$$

This assumes no physical deterioration until half the average use life after which a straight line depreciation occurs. The equation (26) therefore introduces $s = \max_i \{\lambda_i\}$ -- order feedback or "echo" effects into the system. The rows of the constraints defining the feasible region for a given year (4') include rows for each such capacity whose b_{jk} coefficients are positive for each production activity to utilizing the capacity, negative for the investment activity which adds capacity in that given year and zero for activities that do not use or add to that capacity.

3.4.3 Financial Constraints

Four financial constraints indexed by f_1, \dots, f_4 are included in the current model. The first specifies that working capital expended material purchases, machine investment, banking, debt repayment and commercial consumption cannot exceed the amount available which is determined by past sales augmented by current borrowing. Hence, the limitation coefficient of this constraint is:

$$(27) \quad C_{f_1,t} = \sum_{j \in S} a_{j,t-1} X_{j,t-1}^* - \bar{E}_t.$$

The second specifies that borrowing at a relatively low interest rate is limited to a fraction of previous commercial sales, a relationship that crudely approximates the loan practices of bankers. Hence, we have

$$(28) \quad C_{f_2,t} = g \sum_{j \in S} a_{j,t-1} X_{j,t-1}^*.$$

The last two require that loans be repaid (or refinanced annually). Hence

$$(29) \quad X_{jt} \geq (1 + a_{j,t-1}) X_{j,t-1}^*, \quad j = b_1, b_2$$

where b_1, b_2 denote borrowing at a relatively low rate a_{b_1} and at a relatively high rate, a_{b_2} , respectively.

3.4.4 Adoption

Learning new technology is partially based on exposure and which can be measured by the "amount" already adopted. Specifically, we assume that exposure is proportional to use, and that use is measured by the maximum total activity level X_j^{t-1} , already allocated to the new activity in the preceding decade. Let N be the set of "new" activities. Then

$$(30) \quad C_{tj} = (1 + \alpha_j) \max_s \{X_{j,t-s}^*, s = 1, \dots, 10\}, j \in N.$$

This, it must be remembered, gives the maximum expected amount of adoption in the region under conditions favorable to it. If it is currently un-economic, or if other constraints prevent it, adoption in a given year will fall below this amount. The model then explains internally whether or not adoption will proceed according to this maximal rate.

3.3.5 Subsistence Satisficing Constraints

The subsistence requirements are determined exogenously by the number of farm families and by survey data on home consumption as described in § 3.2.1.

3.3.6 Commercial Consumption

Commercial consumption is also determined exogenously from farm family budget data as described above in § 3.2.2.

3.3.7

The first set of safety limitation coefficient have the adaptive form

$$(31) \quad C_{it}^u = (1 + \beta_i^u) \sum_{j \in P_i} X_j^{t-1}, i \in S$$

$$(32) \quad C_{it}^l = - (1 - \beta_i^l) \sum_{j \in P_i} X_j^{t-1}, i \in S,$$

in which P_i is the set of production activities using land to produce commercial crop $i \in S$. The constraints corresponding to (31)-(32) are called flexibility constraints because they describe how flexible a farmer is in any one year in modifying his cropping patterns to take advantage of currently profitable opportunities. An interesting alternative form has been used by CIGNIO [1971].

The second set of safety constraints are based on the old idea that capital stock is adjusted more or less gradually because of the risks involved in immediate adjustment. If C_{it}^* , $i \in M$ is the amount of capital service available in the region in year t of the i^{th} capital good and \bar{C}_{it} the maximum amount that could be used under any condition, then the current maximum investment potential is $\bar{C}_{it} - C_{i,t-1}$, $i \in M$. The adjustment limitation is then

$$(33) \quad C_{it} = \gamma_i [\bar{C}_{it} - C_{i,t-1}], i \in M$$

where γ_j is an adjustment coefficient and to each $i \in M$ there corresponds exactly one $i' \in M'$. Because of the depreciation relation (26) equation relation involves $s+1$ order feedback. These bounds, let it be emphasized, are upper bounds and will be reached only if investment appears to be profitable and if other factors such as learning, financing, labor, etc., are not limiting DAY et. al. [1969].

3.4 The RLP Model

The feedback functions (26)-(33) provide an operational approximation of equation (16). The linear programming problem whose algorithmically selected solution approximates (15) is given by (24). The operational model then consists of a sequence of linear programming problems each one of which is used to estimate production, household, investment and marketing activities in the region for a given year, and the feedback functions which represent how the region's external environment influences farmers' decision problem, how new information is incorporated and how behavioral parameters are adaptively modified on the basis of experience and new conditions.

The imperfections in this operational model are evident and no doubt numerous improvements can and one day should be made. At this point, however, we shall concentrate on a detailed evaluation of the model's ability to track recent history. Our objective is to find out if it can be used -- in its present form -- for projections and policy analysis.

4. MODEL EVALUATION

4.1. Model results

The model was used to simulate regional agricultural history for the period 1952 to 1965. The results can be aggregated to yield a set of variables for which comparable regional data exist. In this set are the acreages sown to various crops over the 14 year period. They also include variables for which no comparable data are available, such as predicted levels of resource use for family labor, hired labor, animal draft and various machine capacities, levels of investments and capacity used of new power sources, levels of production, sales (marketed surplus) and retained consumption of various farm outputs, use of chemical fertilizers by crop and predicted levels of grain sales, working capital used, borrowings at various rates of interest and savings, all on a regional basis. The first set provides the basis for our model evaluation.

Let P_i^t , $i = 1, \dots, q$ be field crop acreage variables for year t . Let P_i^{ot} stand for the "observed" datum and P_i^{mt} stand for the corresponding model variable obtained by aggregating for period t the appropriate regional activity levels X_j^{*t} . We then have two series: P_i^{ot} , $t = 1952, \dots, 1965$; P_i^{mt} , $t = 1952, \dots, 1965$, that may serve as the basis for a model evaluation.

The aggregate series available include irrigated, unirrigated and total crop acreages for the winter (rabi) crops: wheat, gram and barley, and the summer (kharif) crops: cotton, maize, rice, groundnut and bajra (spiked millets), and an annual sugarcane crop that spans both cropping seasons. The several "observed" and model series are displayed graphically in Figure 1 (except for barley whose acreage is insignificant.) "Prediction-

realization" diagrams also provide useful graphs for comparing model and "observed" results. These are displayed in Figure 2 in terms of total field crop acreages by crop.

Various more or less ad hoc statistical methods can be used to measure how well the model captures various specific characteristics of the data. The characteristics we consider are (1) absolute levels, (2) relative variable levels, (3) directions of change and (4) turning points. The ability of the model to "explain" these characteristics in the observed data is compared with a naive model appropriately defined in each case.

4.2 Variable levels

A rough idea of goodness of fit of the model to the observed data levels can be obtained by regressing the "observed" on the model generated variables and computing the associated coefficient of determination (R^2). Results of this type are given in Table 1. A glance at the R^2 column shows that the model explains the acreage levels very well for most crops -- wheat (total and irrigated), cotton (total and American), maize (total and irrigated), rice, groundnut (total, irrigated and unirrigated) and bajra (total); moderately well for two -- wheat (unirrigated) and bajra (unirrigated); and very poorly for barley (unirrigated), maize (unirrigated) and sugarcane. The results for these crops are poor in all respects. The "t" values indicate that the intercept estimates are different from zero at the 5% level of significance only for barley (unirrigated), cotton (D) and sugarcane, and for maize (unirrigated). The "slope" estimates are significantly different from unity for cotton (American), maize (unirrigated), sugarcane, maize (unirrigated) and barley (unirrigated).

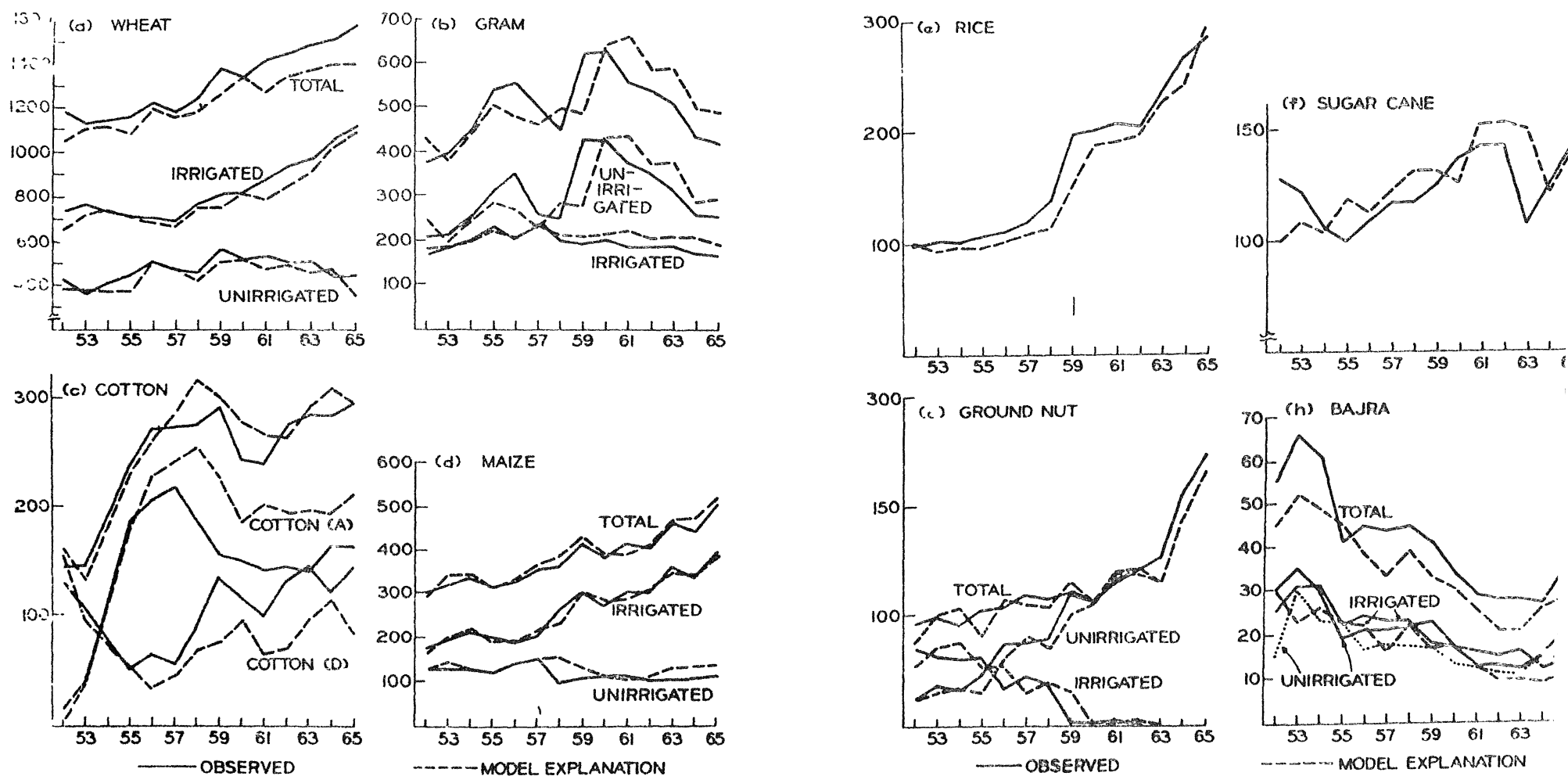


FIG 1: OBSERVED AND MODEL EXPLANATIONS OF FIELD CROP ACREAGE IN THE CENTRAL PUNJAB, 1952-1965
 (THOUSANDS OF ACRES)

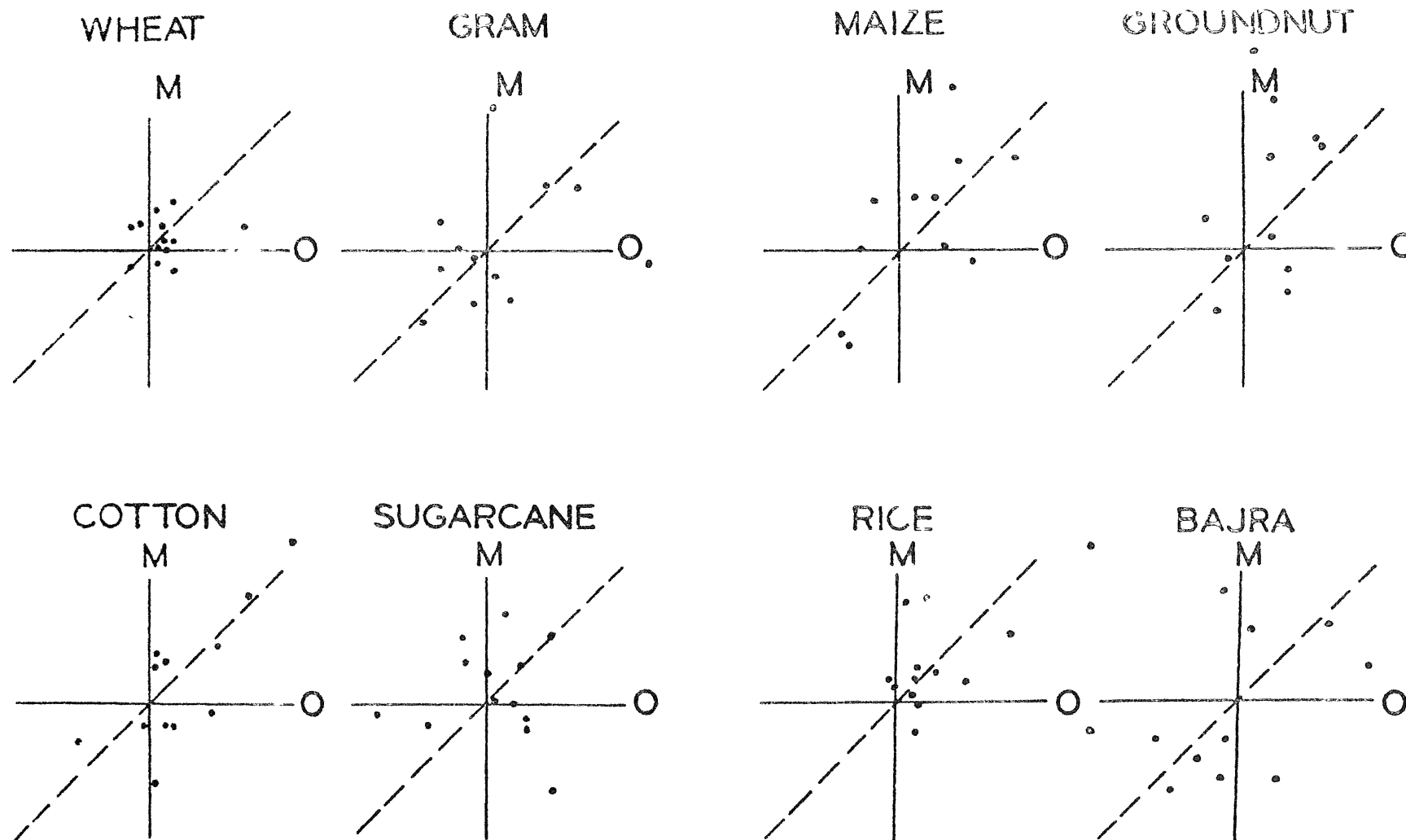


FIG. 2: PREDICTION-REALIZATION DIAGRAMS FOR TOTAL CROP ACREAGES

M-AXIS MEASURES PERCENTAGE ACREAGE CHANGES "PREDICTED" BY THE MODEL
 O-AXIS MEASURES ACREAGE CHANGES "OBSERVED"

TABLE 1: REGRESSION OBSERVED OF MODEL EXPLANATION
OF OBSERVED LEVELS OF FIELD CROP ACREAGES.

Statistic Crop*	Intercept	"t" value t_{α}	Regression Coefficient	"t" value	Coefficient of Variation
Wheat (T)	71.27	0.6054	0.9960	0.0427	.9045
Wheat (I)	25.79	0.4339	1.018	0.2447	.9413
Wheat (U)	100.36	1.4015	0.8354	1.0459	.7012
Gram (T)	168.29	1.5021	0.6447	1.639	.4343
Gram (I)	-12.74	0.2215	0.9960	0.0145	.5219
Gram (U)	101.19	1.5538	0.6666	1.591	.4574
Barley (U)	24.66	6.0049	-.23	5.309	.0759
Cotton (T)	31.02	1.5356	0.8403	2.069	.9081
Cotton (D)	48.35	2.5113	0.6689	1.458	.4197
Cotton (A)	17.028	1.126	0.7227	3.486	.8731
Maize (T)	30.30	1.3384	0.8953	1.8388	.9537
Maize (I)	-3.505	0.2543	1.0233	0.4576	.9711
Maize (U)	73.72	2.0218	0.3305	2.4046	.1051
Rice (I)	13.92	1.3266	0.9973	0.0447	.9564
Sugarcane (T)	73.93	2.7936	0.3867	2.9722	.2264
Groundnut (T)	-0.94	0.7607	1.0533	0.5672	.9129
Groundnut (I)	-0.60	0.012	0.914	0.6878	.8168
Groundnut (U)	3.028	0.5721	1.0548	1.1804	.9773
Bajra (T)	1.734	0.3848	1.1323	1.0676	.8743
Bajra (I)	6.138	1.4397	0.8302	0.7171	.5058
Bajra (U)	6.397	1.8282	0.8166	0.9875	.6171

*T = Total; I = Irrigated; U = Unirrigated

Serious objections to this method of evaluation can be raised: the model estimates are not independent while the tests assume they are; and, the test takes no account of the relative importance of the variables. The first objection vitiates the theory of significance lying behind the t ratios. Hence, at best the statistics of Table 1 must be regarded as informal measures of goodness of fit and model bias, that tend to over-estimate model error. Nonetheless they are effective in a descriptive way, and on the basis of them we gain the impression that the Punjab model is fairly effective at estimating field crop levels, though not with great precision.

4.3. Relative variable levels

Both the interdependence of the estimates and a weighting according to magnitude are incorporated into the Information Inaccuracy statistics introduced into econometric work by Theil and various of his collaborators, e.g. THEIL [1967], TILANUS and THEIL [1965]. We have computed the average information inaccuracy, the expected information content, the relative information inaccuracy for all field crops and have compared the results with those that are obtained from a naive model in which the proportion of land devoted to a given crop is predicted to be the same as in the previous year. These statistics are given in Table 2.

TABLE 2: INFORMATION STATISTICS FOR FIELD CROP SHARES

(1) YEAR	ALL FIELD CROPS			RABI CROPS	KHARIF CROPS
	(2) INFORMATION INACCURACY	(3) EXPECTED INFORMATION CONTENT	(4) RELATIVE INFORMATION INACCURACY	(5) RATIO OF RELATIVE INFORMATION INACCURACY MODEL ÷ NAIVE	(6) RATIO OF RELATIVE INFORMATION INACCURACY MODEL ÷ NAIVE
1952	.006682	1.620	.004123	N.A.	N.A.
1953	.001771	1.644	.001077	0.2606	3.0346
1954	.001682	1.648	.001021	0.0832	0.3666
1955	.002866	1.642	.001758	0.1255	0.2149
1956	.003037	1.651	.001839	0.2214	0.0352
1957	.005032	1.699	.003016	0.4414	0.0936
1958	.006215	1.664	.003735	1.2407	0.2079
1959	.00968	1.680	.005763	0.6049	0.1709
1960	.001575	1.665	.000946	0.0545	0.0885
1961	.006586	1.655	.00398	1.3389	0.0823
1962	.003396	1.657	.002049	1.2023	0.0374
1963	.008084	1.661	.004867	1.7161	0.1418
1964	.002694	1.665	.001618	0.2697	0.0475
1965	.002505	1.688	.001484	0.4852	0.0202
Average	.003794	-	.002406	0.6484	0.1255

Although no level of significance can be assigned to these non-parametric statistics, it is obvious that the model predicts the proportions quite well. In no year does the relative information loss in the model exceed 0.6 percent, while on the average the model loses less than 0.3 percent of the information contained in the observed proportions. Moreover, on the average the simulation model is seen to out perform the naive model about 1 1/2 times for rabi season crops. It out performs the naive model 8 times for kharif season crops.

4.4. Directions of change and turning points

Correct and incorrect model explanations of qualitative events should be weighted for the same reason that proportionate levels of variables are weighted in the information inaccuracy statistic: some predictions possess a great deal of information but are rare events while others possess very little information but are frequent events. THEIL [opt. cit., pp. 12, 31, 34] suggests the mutual information or the information gain or loss as a measure that involves a suitable weighting of the information contained in the explanation-observation table. Routine manipulation of his equation for mutual information enables one to identify three components of information about the qualitative performance of the model. These are the observed information, O , the model information, M , and the joint information, J , respectively. Each of these can be decomposed into explained and unexplained or true and false components. Hence we have O_T , O_F , M_T , M_F , J_T and J_F .

Each of these measures can be computed for the RLP model and for any alternative model. For purposes of comparison here we have used a naive

model that predicts a direction of change to be the same as the preceding period, and for turning points, one that predicts no turning point every period. Tables 3 and 4 present the results of this exercise.

TABLE 3: QUALITATIVE INFORMATION STATISTICS FOR DIRECTIONS OF CHANGE EXPLAINED BY RLP MODEL AND NAIVE ALTERNATIVE

	OBSERVED INFORMATION EXPLAINED O_T/O			CORRECT MODEL INFORMATION M_T/M			CORRESPONDING JOINT INFORMATION J_T/J		
	RLP	NAIVE	RATIO	RLP	NAIVE	RATIO	RLP	NAIVE	RATIO
Wheat	.51	.24	2.15	.51	.30	1.70	.46	.33	1.41
Gram	.50	.76	.66	.50	.75	.67	.50	.59	.85
Barley	.53	.24	2.23	.53	.25	2.14	.51	.41	1.24
Cotton (D)	.53	.41	1.29	.53	.42	1.28	.51	.48	1.07
Cotton (A)	.43	.49	.88	.52	.59	.89	.44	.47	.94
Maize	.66	.21	3.11	.72	.21	3.37	.55	.33	1.64
Rice	.27	.27	1.00	.27	.27	1.00	.31	.31	.98
Cane	.33	.43	.77	.33	.41	.79	.36	.40	.91
Groundnut	.68	.46	1.47	.61	.46	1.33	.54	.46	1.16
Bajra	.62	.16	3.79	.76	.16	4.64	.53	.25	2.14

First consider directions of change. In seven of ten cases at least half the observed information was explained and in eight or ten cases more than half the model information was correct. In about half the cases the RLP model out performs the naive alternative substantially. This is particularly impressive in view of the strong trend in most of the crops that tend to favor the naive alternative. It is interesting to note that more than half the information on directions of change for barley was correctly "explained" by the model even though only 7% of the variance in crop acreages was explained.

In the case of turning points Table 6 shows that the RLP model out performs the naive model in about half the cases, explaining from a mere 7% of the observed information to 71% of the joint information for bajra. The naive model explains about a third of the information in every case. These results are mixed but suggest that turning point predictions are difficult to explain even when direction of change, levels and proportions are tracked fairly well.

TABLE 4: QUALITATIVE INFORMATION STATISTICS
FOR TURNING POINTS EXPLAINED BY RLP MODEL
AND NAIVE ALTERNATIVE

	OBSERVED INFORMATION EXPLAINED O_T/O			CORRECT MODEL INFORMATION $M_T/M^{(a)}$			CORRESPONDING JOINT INFORMATION J_T/T		
	RLP	NAIVE	RATIO	RLP	NAIVE	RATIO	RLP	NAIVE	RATIO
Wheat 1/2 (b)	.46	.33	1.41	.44			.41	.33	1.25
Gram 1/6	.07	.30	.22	.17			.20	.30	.66
Barley 1/6	.15	.32	.47	.19			.21	.32	.65
Cotton (D) 5/12	.36	.34	1.05	.33			.32	.34	.95
Cotton (A) 5/12	.19	.31	.62	.28			.23	.31	.74
Maize 7/12	.58	.33	1.75	.67			.53	.33	1.60
Rice 7/12	.27	.31	.88	.23			.23	.31	.75
Cane 4/12	.39	.30	1.29	.33			.34	.30	1.13
Groundnut 5/12	.44	.33	1.33	.42			.42	.33	1.27
Bajra 8/12	.61	.33	1.83	.71			.55	.33	1.65

(a) Naive model contains no information about turning points by definition. Hence ratio is always infinite.

(b) Numbers following crop names are the ratios $\sum_i f_{ii}$.

4.5 Summary of the model evaluation

Enough evidence has now been accumulated to obtain a good impression of how well our model captures reality at least so far as recent history in the Punjab goes. It appears that (1) the model fairly accurately explains

levels of field crop acreages; (2) it explains quite well the pattern of cropping in the region from year to year; (3) it explains directions of change with some -- perhaps surprisingly great -- accuracy; (4) it explains turning points only modestly on a year to year basis. In addition to the quantitative measures summarized above the model presents a qualitative picture of development in close accord with general descriptive characterization of the region's recent history. This can be seen in detail in two papers presented by us elsewhere. SINGH and DAY [1972A] and SINGH and DAY [1972B].

For our own part, we believe the evidence supports the inference that our model captures a significant part of the structure of the agricultural economy of the Punjab; that it supports the theory of farm decision making presented in this paper; and while scarcely an accurate predictor of annual events and while clearly leaving plenty of room for improvements, it is good enough to use now both for gaining a clearer understanding of past development and for projecting likely future developments under presently conceived policy alternatives. We have reported our applications for both these purposes in the two references just cited.

NOTES

1. The argument of this section was first presented by us at the seminar of Professors NAKAJIMA and MARUYAMA at Kyoto University in October, 1966.
2. The list is long. The following are representative references: BOEKE [1953], DABASI-SCHWENG [1965], DALTON [1962], FUSFIELD [1957], LEWIS [1955], NAIR [1965], NEAL [1959], OLSON [1960], WHARTON [1963]. Apparently ignorant of or immune to the flood of econometric evidence in the meantime MYRDAL [1968] joined this "traditionalist" school with a vengeance.
3. These studies include those of BAUER and YAMEY [1959], BEHRMAN [1967a], [1967b] and [1968], BROWN [1963], DEAN [1965], FALCON [1964], KAUL [1967], KRISHNA [1963], MANGAHAS [1966], MUYBARTO [1965], and STERN [1962].
4. The sequence (Φ_1, \dots, Φ_4) is called a lexicographic* or a L^* utility function. Cf. ENCARNACION [1964a], [1964b], ROBINSON and DAY [1971]. Cf. also CHIPMAN [1960], FERGUSON [1965], GEORGESCU-ROEGEN [1954].
5. Theoretically our procedure has the following interpretation. We assume that a well defined utility function, Φ_2 , exists whose upper contour sets are convex. Moreover, in this initial study we assume that this function is satiated for all years included. Hence the set of household activities satisfying the subsistence goal can be approximated by a polyhedron defined by the linear inequalities

$$\tilde{H}(B^S, C_t^S) = \{X \mid \sum_{j \in H} b_{ij} X_j \leq C_{it}, i \in E\}$$

where H is the set of household activities and E the set of approximating hyperplanes. These describe how satisfaction of anticipated subsistence consumption requirements can be met by planning for adequate amounts of commercial purchases or by using up enough farm produced commodities. In theory the coefficients b_{ij} , c_i , $i \in E$, $j \in H$, depend at the microlevel on the a^{it} vector and σ_i^t parameter of the utility function Φ_1 of equation (5).

6. Our colleague, Mohinder S. Mudahar, is currently experimenting with an endogenously incorporated cash consumption function that allocates current cash to consumption on the basis of lagged cash income and the lagged internal rate of return on capital. The former variable depends on lagged sales activities and the latter on the lagged shadow price on working capital. Such a relation can be derived from a utility function as required by our theory by using the notion of flexible assets KOOPMANS [1964] as shown by DAY [1969]. Elsewhere it is shown that such a function can generate golden rule growth paths in a one sector growth model DAY and FAN [1971], though their theoretical properties in the present more complicated model are not yet known.

7. The general reasoning behind such safety constraints is elaborated in DAY [1970b] and DAY [1971]. Alternative versions of this method of accounting for uncertainty include the chance constrained programming of CHARNES and COOPER [1959], the Safety-First Principle of ROY [1952] and the Focus Loss Principle of SHACKLE [1958]. The last principle has been applied by PETIT and BOUSSARD [1967]. Comparison of these methods with the conventional portfolio approach FREUND [1956] has been made by BOUSSARD [1969]. We use here the form suggested by HENDERSON [1959] cf. below § 2.3 (7).

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